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Analysis of Steady Blood Flow with Casson Fluid along an Inclined Plane Influenced by the Gravity Force

Pradeep Porwal and V.H. Badshah
School of Studies in Mathematics,
Vikram University, Ujjain (M.P.), India

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ABSTRACT: In this paper, we have considered a Non-Newtonian Casson fluid model of blood flow and obtained the expression for velocity and flow rate. We have investigated some characteristics of steady one dimensional laminar blood flow along an inclined plane influenced by the gravitational force. The velocity profile and flow rate were discussed graphically for various values of yield stress β and inclination angle θ .

Keywords: Blood flow, Casson fluid model, velocity Profile, flow rate, hematocrit.
AMS Classification: 76Z05, Secondary 92C35, 76ZXX.

I. INTRODUCTION

Blood is essential for human life and plays a number of roles in the body. Human blood is a complex fluid consisting of a suspension of cell in an aqueous, fibreless and yellow colored solution called plasma. Blood cells generally known as the formed elements of three types, i.e. red blood corpuscle (erythrocytes), white blood cells (leukocytes) and platelets. The flow of blood is that it exhibits a yield stress. To explain the behavior of blood in different theoretical models are proposed i.e. Newtonian and Non-Newtonian.

Chaturani and Upadhyay [8] have been studied the gravity flow with couple stress along an inclined plane.

Verma [7] proved the theoretical model to study of power law fluid flow along an inclined plane influenced by the gravity force with application to blood flow.

Bugliarello and Sevila [4] and Coklet [5] indicated that under certain flow conditions blood flow may deviate significantly from the Newtonian behavior.

Han and Barnett [2] and Whitemore [6] have been reported that the suspension of red cells in serum, the behavior of which is similar to the whole blood for shear rates more than 1/sec obey power law model.

In this present paper, we have considered the gravity flow of blood through small diameter tube satisfying Casson constitutive equation of motion along an inclined plane. Velocity profile, flow rate and pressure distribution are obtained. Results are discussed for different values of β and inclination angle θ.

II. MATHEMATICAL FORMULATION AND ANALYSIS

We consider the steady one dimensional laminar blood flow along an inclined plane at an angle θ with the horizontal axis. It is assumed that blood is incompressible and characterized by the Casson fluid model. It is also assumed that there is no pressure gradient in x-direction and thickness of blood layer is ‘a’ which is uniform throughout the plate. Also assume that the velocity gradient is zero at y = a and only body force is gravity.

The constitutive equation for Casson fluid model is given by

$$\tau^{1/2} = \mu^{1/2} . e^{1/2} + \tau_0^{1/2} , \tau \geq \tau_0$$

$$\text{And } e = 0, \qquad \tau < \tau_0 \qquad ...(1)$$

Where τ is shear stress, μ is coefficient of viscosity e is velocity gradient and τ_0 is yield stress.
For steady one dimensional gravity flow velocity field is given by:

$$sV_x = V(y),\ V_y = V_z = 0 \qquad \dots (2)$$

and $$\frac{\partial p}{\partial x} = 0, \qquad \dots(3)$$

where p is pressure.
Equations of motion under above assumption along the x-direction are:

$$\frac{\partial \tau}{\partial y} + \rho g x = 0 \qquad \dots (4)$$

$$-\frac{\partial p}{\partial y} + \rho g y = 0 \qquad \dots (5)$$

$$-\frac{\partial p}{\partial z} = 0 \qquad \dots (6)$$

Gravitational component in x and y directions are:

$$g_x = g\sin\theta \text{ and } g_y = -g\cos\theta$$

Then equation (4), (5) and (6) becomes

$$\frac{\partial \tau}{\partial y} + \rho g \sin\theta = 0 \qquad \dots (7)$$

$$-\frac{\partial p}{\partial y} - \rho g \cos\theta = 0 \qquad \dots (8)$$

$$-\frac{\partial p}{\partial z} = 0 \qquad \dots (9)$$

Where g is acceleration due to gravity and ρ is the density of blood.
The boundary conditions are:
at $y = 0$, $V = 0$ … (10)
at $y = a$, $p = p_0$,

$$\frac{dV}{dy} = 0,\ \tau_0 = 0 \qquad \dots (11)$$

The solution of equation (8) is given by

$$p = p_0 + \rho g(a - y)\cos\theta \qquad \dots (12)$$

Where p_0, the atmospheric pressure and p is the pressure distribution in y-axis.
The solution of equation (7) with boundary condition (10) and (11) is:

$$V = \left(\frac{\rho g \sin\theta}{\mu}\right).\frac{a^2}{6}\left\{6\delta(1+\beta\cos ec\theta) - 3\delta^2 + 4(\beta\cos ec\theta)^{\frac{1}{2}}.(4-3\delta)\right\} \qquad \dots (13)$$

Where parameter $\beta = \dfrac{\tau_0}{\rho g a}$ and $\delta = \dfrac{y}{a}$.

The flow rate is defined as

$$Q = \int_0^a V dy \qquad \dots(14)$$

Using equation (13) and (14) we obtained

$$Q = \left(\frac{\rho g \sin\theta}{\mu}\right).\frac{a^3}{6}\left[2 + 3\beta\cos ec\theta + 8\left(\beta\cos ec\theta\right)^{\frac{1}{2}}\right] \qquad \dots(15)$$

III. RESULT AND DISCUSSION

The viscosity of normal blood have calculated from the relation given by Gupta [1] $\mu = \mu_p (1 + 0.025H + 7.35 \times 10^{-4} H^2)$

Where μ_p is the viscosity of plasma, H is the hematocrit of the blood. The plasma viscosity $\mu_p = 0.012 = 1.2 \times 10^{-3}$ Ns/m^2 by Merrill [3]. We have calculate the viscosity of normal blood $\mu = 0.038$ poise $= 3.8 \times 10^{-8}$ Ns/m^2 for 40% hematocrit. It is considered that the distance between planes a = 200 μm = 2×10^{-4} m for microcirculatory system, density of blood ρ = 1060 kg/m^3, g = 9.81 m/sec^2 and τ_0 = 0.005 N/m^2. We have calculate the non-dimensional yield stress parameter $\beta = 0.0024$.

Using these data the variation of velocity profile with angle of inclination θ are shown in fig. (1) and (2). From these figures and equation (16) it is clear that the flow is maximum for $\theta = 90^0$ and no flow occur for $\theta = 0^0$ i.e. V = 0. From figure it is observe that velocity at any point in the layer increases with θ. Fig. (1) is the variation of velocity with δ and for different angle of inclination for β = 0.00 (power law fluid). Fig. (2) is the variation of velocity with δ for different angle of inclination with β = 0.0024 (Casson fluid model).

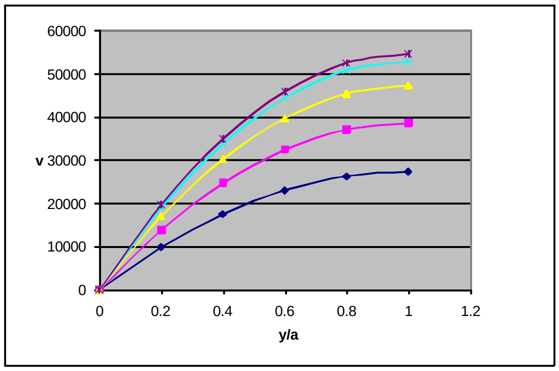

Fig. 1. Variation of velocity with y/a = δ for different angle of inclination and for β = 0.00.

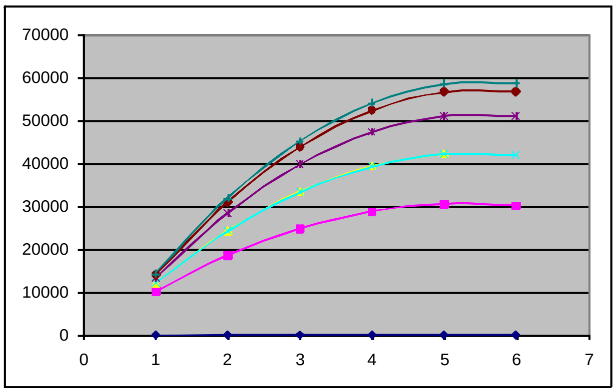

Fig. 2. Variation of velocity with y/a = δ for different angle of inclination and for β = 0.0024.

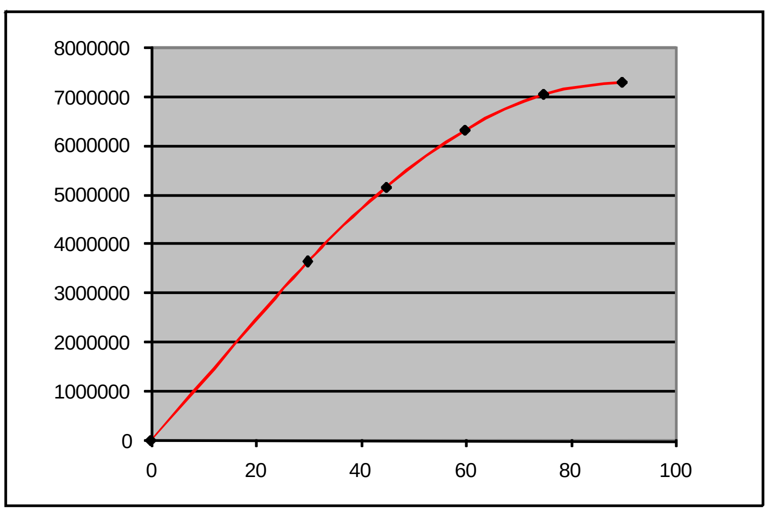

Fig. 3. Variation of flow rate with different angle of inclination and for β = 0.00.

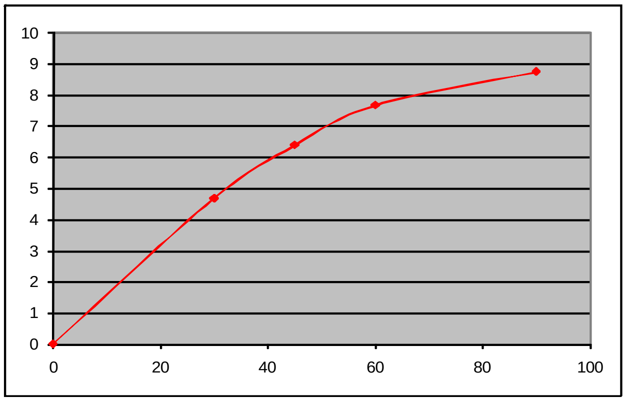

Fig. 4. Variation of flow rate with different angle of Inclination and for β = 0.0024.

Fig. (3) and (4) are the variation of flow rate with angle of inclination for $\beta = 0$ and $\beta = 0.0024$ respectively.

From these figures it is observed that numerical value of velocity increasing with increasing angle of inclination.

From equation (13) non-dimensional pressure is obtained as :

$$\bar{P} = \left(1 - \frac{y}{a}\right) \cos\theta ,$$

Where, $\bar{P} = \dfrac{p - p_0}{\rho g a}$.

The pressure distribution with δ for different angle β is shown in below table.

θ \ δ	0.00	0.20	0.40	0.60	0.80	1.00
0^0	1.000	0.800	0.600	0.400	0.200	0.000
30^0	0.866	0.693	0.520	0.346	0.173	0.000
45^0	0.707	0.566	0.424	0.283	0.141	0.000
60^0	0.500	0.400	0.300	0.200	0.100	0.000
75^0	0.259	0.207	0.155	0.104	0.052	0.000
90^0	0.000	0.000	0.000	0.000	0.000	0.000

From above table, we see that the pressure decreases with increasing δ and θ is maximum when $\theta = 0^0$ and $\delta = 0$ at lower plane.

The variation of flow rate and velocity with angle of inclination θ and yield stress parameter β is shown in tables 1 and 2.

Table 1. Variation of flow rate Q with β = 0.00 and β = 0.0024.

θ	For $\beta = 0.00$	For $\beta = 0.0024$
0^0	0	0
30^0	3.6485×10^6	4.685×10^6
45^0	5.1605×10^6	6.388×10^6
60^0	6.3194×10^6	7.675×10^6
75^0	7.0484×10^6	8.479×10^6
90^0	7.2973×10^6	8.752×10^6

From Table 1, we see that the flow rate increases with angle of inclination θ and from equation (17) there is no flow when $\theta = 0^0$.

Table 2. Variation of velocity V with β = 0.00.

θ \ δ	30^0	45^0	60^0	75^0	90^0
0.2	9849.6	13926.6	17059.6	19028.5	19699.2
0.4	17510.4	24758.6	30328.3	33828.5	35020.8
0.6	22982.4	32495.4	39805.9	44399.9	45964.8
0.8	26265.6	37137.6	45492.5	50742.7	52531.2
1.0	27360	38685	47388	52857	54720

Table 3. Variation of velocity V with β = 0.0024.

δ \ θ	30^0	45^0	60^0	75^0	90^0
0	10104.96	12018.14	13268.64	14024.72	14300.16
0.2	18491.71	24193.6	28375.93	31002.04	31906.87
0.4	24689.66	33274.26	39692.19	43750.8	45135.97
0.6	28698.82	39260.12	47217.4	52270.99	53987.48
0.8	30519.17	42151.18	50951.58	56562.63	58461.39
1	30150.72	41947.44	50894.71	56625.7	58557.7

Table 2, 3 are the variation of velocity with δ for different angle of inclination and for $\beta = 0.00$. From this table we see that the velocity is increases with increasing angle of inclination θ and δ.

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